Answer on Question #68659 - Math – Other

Question:

Find the coefficient of x^{16} in the expression of $(x^2 - 2x)^{10}$.

Solution:

$$(x^2 - 2x)^{10} = \big(x(x-2)\big)^{10} = x^{10}(x-2)^{10} = x^{10}\sum_{k=0}^{10}\binom{10}{k}x^{10-k}(-2)^k$$

The coefficient of x^{16} in the whole expression is the coefficient of x^6 in $\sum_{k=0}^{10}\binom{10}{k}x^{10-k}(-2)^k$, which is when $k = 4$:

$$c = \binom{10}{4}(-2)^4 = \frac{10!}{4!\,6!}(-2)^4 = \frac{10\times9\times8\times7}{1\times2\times3\times4}\times16 = 210\times16 = 3360$$

Answer:

3360