Answer on Question #68577 - Math - Linear Algebra

Let $V$ be the subspace of $P_3$ spanned by the the following set

$$\{1 - x^2 + x^3,\ \ 2 + x - x^2 + x^3,\ \ 1 + 2x + x^2 - x^3\}$$

a) Show that $f(x) = x + x^2 - x^3 \in V$.
b) Show that $g(x) = 1 + x - x^2 + x^3$ is not an element of $V$.
c) Find a basis for $V$ which contains $f(x)$.
d) Find a basis for $P_3$ which contains $g(x)$.

## Solution:

a)

$$[[v_1]\quad [v_2]\quad [v_3]\quad [f]] = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -2 & -1 \end{bmatrix} \to$$

$$\to \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Column $[f]$ is not a pivot column, so $f(x) \in V$

b)

$$[[v_1]\quad [v_2]\quad [v_3]\quad [g]] = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ -1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix} \to$$

$$\to \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Column $[g]$ is a pivot column, so $g(x) \notin V$

c) Columns $[f]$ and $[v_1]$ are pivot columns of the matrix $[[f]\quad [v_1]\quad [v_2]\quad [v_3]]$. So $f(x)$ and $v_1(x)$ form a basis for V.

d) Any three vectors of the standard basis together with $g(x)$ form a basis for $P_3$.