Answer on Question #68401 – Math – Differential Equations

Question

Obtain the partial differential equation by eliminating the arbitrary constant from the relation

$$u = xy + y\sqrt{x^2 - a^2} + b$$

Solution

$u = xy + y\sqrt{x^2 - a^2} + b$

$p = \frac{\partial u}{\partial x} = y + \frac{xy}{\sqrt{x^2 - a^2}}$

$q = \frac{\partial u}{\partial y} = x + \sqrt{x^2 - a^2}$

$\sqrt{x^2 - a^2} = q - x \to p = y + \frac{xy}{q - x}$

$\frac{\partial u}{\partial x} = y + \frac{xy}{\frac{\partial u}{\partial y} - x}$ is a partial differential equation.

Answer: $\frac{\partial u}{\partial x} = y + \frac{xy}{\frac{\partial u}{\partial y} - x}$.