Answer on Question #68369 – Math – Differential Equations

Question

Form a partial differential equation of

$$u = ae^{by} + b\sin bx$$

Solution

If

$$u = ae^{by} + b\sin bx,$$

then

$$\frac{\partial u}{\partial x} = b^2\cos bx,$$
$$\frac{\partial u}{\partial y} = abe^{by},$$
$$\frac{\partial^2 u}{\partial x^2} = -b^3\sin bx,$$
$$\frac{\partial^2 u}{\partial y^2} = ab^2e^{by}.$$

Thus, the function

$$u = ae^{by} + b\sin bx$$

satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + b^2u = 0.$$

On the other hand, the function

$$u = ae^{by} + b\sin bx$$

satisfies the system of partial differential equations:

$$\begin{cases} \dfrac{\partial u}{\partial x} = b^2\cos bx, \\ \dfrac{\partial u}{\partial y} = abe^{by}. \end{cases}$$