Answer on Question #68368 – Math – Differential Equations

Question

Form Partial differential eq. of u=xy+y√(x^2-a^2)+b

Solution

$$u = xy + y\sqrt{x^2 - a^2} + b$$
$$p = \frac{\partial u}{\partial x} = y + \frac{xy}{\sqrt{x^2 - a^2}}$$
$$q = \frac{\partial u}{\partial y} = x + \sqrt{x^2 - a^2}$$
$$\sqrt{x^2 - a^2} = q - x \ \rightarrow p = y + \frac{xy}{q - x}$$

A partial differential equation (PDE) is

$$p = y + \frac{xy}{q-x},$$

that is,

$$\frac{\partial u}{\partial x} = y + \frac{xy}{\frac{\partial u}{\partial y} - x}$$

Answer: $p = y + \frac{xy}{q-x}$ or $\frac{\partial u}{\partial x} = y + \frac{xy}{\frac{\partial u}{\partial y} - x}$.