Answer on Question #68130 – Math – Linear Algebra

Question

If A, B, C are three vectors, show that $A \cdot (B + C) = A \cdot B + B \cdot C$.

Solution

The given property is not valid. Let's take for example

$$A = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Then we obtain

$$B + C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \qquad A \cdot (B + C) = 2 \cdot 1 + 0 \cdot 2 = 2$$

and

$$A \cdot B = 2 \cdot 0 + 0 \cdot 1 = 0, \qquad B \cdot C = 0 \cdot 1 + 1 \cdot 1 = 1, \quad A \cdot B + B \cdot C = 1.$$

And we see, that $A \cdot (B + C) \neq A \cdot B + B \cdot C$.

But for three arbitrary vectors A, B, C the distributive property of dot product over addition is valid, namely

$$A \cdot (B + C) = A \cdot B + A \cdot C.$$

We now prove it. Let

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \qquad C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}.$$

Then

$$B + C = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} b_1 + c_1 \\ b_2 + c_2 \\ \vdots \\ b_n + c_n \end{bmatrix},$$

$$A \cdot (B + C) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \cdot \begin{bmatrix} b_1 + c_1 \\ b_2 + c_2 \\ \vdots \\ b_n + c_n \end{bmatrix} = a_1 \cdot (b_1 + c_1) + a_2 \cdot (b_2 + c_2) + \cdots + a_n \cdot (b_n + c_n)$$
$$= a_1 \cdot b_1 + a_1 \cdot c_1 + a_2 \cdot b_2 + a_2 \cdot c_2 + \cdots + a_n \cdot b_n + a_n \cdot c_n$$
$$= (a_1 \cdot b_1 + a_2 \cdot b_2 + \cdots + a_n \cdot b_n) + (a_1 \cdot c_1 + a_2 \cdot c_2 + \cdots + a_n \cdot c_n) = A \cdot B + A \cdot C.$$

The distribution property for dot product of vectors is proved.

The cross product is distributive over addition as well, namely for three arbitrary vectors A, B, C

$$A \times (B + C) = A \times B + A \times C.$$

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