Task 1. *Show that a set can't be a neighborhood of its supremum.*

Solution. Suppose that a set $X \subset \mathbb{R}$ is a neighborhood of $x \in \mathbb{R}$. By definition this means that there is an open set $G \subset X$, which contains x. Therefore, $x \in (x-\varepsilon, x+\varepsilon) \subset G \subset X$ for some $\varepsilon > 0$, where $(x-\varepsilon, x+\varepsilon)$ denotes the open interval with bounds $x-\varepsilon$ and $x+\varepsilon$. Hence, X contains the elements, which are strictly greater, than x, for example, the elements from $(x, x+\varepsilon)$. Thus, x cannot be a supremum of X. $\square$