

Answer on Question #67847 – Math – Statistics and Probability

Question

Ten leaves were randomly selected from each of ten similar strawberry trees for a total of 100 leaves. The number of adult female spider mites on each was counted.

Mites/Leaf Frequency

0	16
1	71
2	10
3	3

Does the assumption of a Poisson distribution seem appropriate as a model for these data?

Solution

The mean of the assumed Poisson distribution is unknown so it must be estimated from the data by the sample mean:

$$\mu = \frac{1}{100}(16 \cdot 0 + 71 \cdot 1 + 10 \cdot 2 + 3 \cdot 3) = 1.$$

Using the Poisson distribution with $\mu = 1$ we can compute the expected values:

$$E_0 = 100P(X = 0) = e^{-1} \frac{1^0}{0!} = 36.8;$$

$$E_1 = 100P(X = 1) = e^{-1} \frac{1^1}{1!} = 36.8;$$

$$E_2 = 100P(X = 2) = e^{-1} \frac{1^2}{2!} = 18.4;$$

$$E_3 = 100P(X = 3) = e^{-1} \frac{1^3}{3!} = 6.1.$$

Now we should use the chi-squared goodness of fit test.

Null hypothesis H_0 : observed distribution is consistent with the Poisson distribution.

Alternative hypothesis H_a : observed distribution is not consistent with the Poisson distribution.

Test statistic:

$$\chi^2 = \sum \frac{(E_i - O_i)^2}{E_i} = \frac{(36.8 - 16)^2}{36.8} + \frac{(36.8 - 71)^2}{36.8} + \frac{(18.4 - 10)^2}{18.4} + \frac{(6.1 - 3)^2}{6.1} = 59.8.$$

Degrees of freedom: $df = 4 - 1 = 3$.

P-value: $p < 0.0001$.

Since P-value is less than 0.05 we should reject the null hypothesis and conclude that the Poisson distribution does not seem appropriate as a model for these data.

Answer: the Poisson distribution does not seem appropriate as a model for these data.