Answer on Question #67830 – Math – Linear Algebra

Question

1) Given that

$$a_1 = 2i - j + k$$

$$a_2 = i + 3j - 2k$$

$$a_3 = 3i + 2j + 5k$$

$$a_4 = 3i + 2j + 5k$$

find scalars a, b, c such that

$$a_4 = aa_1 + ba_2 + ca_3$$

Solution

If $a_4 = aa_1 + ba_2 + ca_3$, then

$$3i + 2j + 5k = a(2i - j + k) + b(i + 3j - 2k) + c(3i + 2j + 5k)$$
$$3i + 2j + 5k = (2a + b + 3c)i + (-a + 3b + 2c)j + (a - 2b + 5c)k$$

$$\begin{cases} 2a + b + 3c = 3 \\ -a + 3b + 2c = 2 \\ a - 2b + 5c = 5 \end{cases}$$

It follows from the third equation that

$$a = 5 + 2b - 5c \quad (1)$$

Add the second and the third equations

$$b + 7c = 7,$$

hence

$$b = 7 - 7c \quad (2)$$

Substitute (2) into (1)

$$a = 5 + 2b - 5c = 5 + 2(7 - 7c) - 5c = 5 + 14 - 14c - 5c = 19 - 19c,$$

that is,

$$a = 19 - 19c \quad (3)$$

Substitute (2) and (3) into the first equation of the system

$$2a + b + 3c = 3$$
$$2(19 - 19c) + 7 - 7c + 3c = 3$$
$$38 - 38c + 7 - 7c + 3c = 3$$
$$-42c = -42$$

Hence

$$c = 1 \quad (4)$$

Substitute (4) into (2) and (3)

$b = 7 - 7c = 7 - 7 \cdot 1 = 7 - 7 = 0$

$$a = 19 - 19 \cdot 1 = 19 - 19 = 0$$

Finally one gets

$$a = b = 0 \ ; \ c = 1$$

Answer: $a = b = 0 \ ; \ c = 1$

Question

2) If a and b are non-collinear vectors and $A = (x + y)a + (2x + y + 1)b$

Answer: the statement of the question is not complete and it is not known what one should calculate there.

Question

3) Given the scalar defined by $\phi(x, y, z) = 3x^2 - xy^2 + 5$

Answer: the statement of the question is not complete and it is not known what one should calculate there.