Answer on Question #67801 – Math – Statistics and Probability

Question

The proportion of male and female students in a class is found to be 1 : 2. What is the probability that out of 4 students selected at random with replacement

(i) 2 or more will be females

(ii) no male student is selected ?

Solution

We do not know how many students in total there are in the class. Let x be the number of male students. Then the number of female students in the class is $2x$ and total number of the students in the class is $3x$. Therefore, the probability for a randomly chosen student with replacement to be female is

$$p = \frac{2x}{3x} = \frac{2}{3}.$$

Number of students selected at random with replacement $n = 4$. We have Bernoulli trials process with $n = 4$ and $p = \frac{2}{3}$.

The probability of getting exactly k successes in n trials is given by

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}.$$

(i) We have to find
$$P(2 \text{ or more females}) = P(2) + P(3) + P(4).$$
But it is easier to calculate probability for the complementary event:
$$P(\text{less then 2 females}) = P(0) + P(1) = \binom{4}{0} \left(\frac{2}{3}\right)^0 \left(1 - \frac{2}{3}\right)^4 +$$
$$+ \binom{4}{1} \left(\frac{2}{3}\right)^1 (1 - \frac{2}{3})^3 = \frac{1}{81} + 4 \cdot \frac{2}{3} \cdot \frac{1}{27} = \frac{1}{81} + \frac{8}{81} = \frac{1}{9} \approx 0.11.$$
Then
$$P(2 \text{ or more females}) = 1 - P(\text{less then 2 females}) = 1 - \frac{1}{9} = \frac{8}{9} \approx$$
$$\approx 0.89.$$

(ii) The event 'no male student is selected out of 4 students' is the same as '4 female students is selected out of 4 students'. The corresponding probability is
$$P(4) = \binom{4}{4} \left(\frac{2}{3}\right)^4 \left(1 - \frac{2}{3}\right)^0 = \frac{16}{81} \approx 0.20.$$

Answer: (i) $\frac{8}{9} \approx 0.89$; **(ii)** $\frac{16}{81} \approx 0.20.$