

Answer on Question #67672 – Math – Statistics and Probability

Question

An insurance company checks police records on 593 accidents selected at random and notes that teenagers were at the wheel in 83 of them.

a) Construct the 95% confidence interval for the percentage of all auto accidents that involve teenage drivers.

Solution

If $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$, we can use the following formula to compute the confidence interval for the true population proportion:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}},$$

where \hat{p} is the sample proportion, n is the sample size, z^* is multiplier that dependent on the level of confidence:

Confidence Level	z^* multiplier
90%	1.645
95%	1.960
98%	2.326
99%	2.578

In our case, we have $\hat{p} = \frac{83}{593} \approx 0.14$, $n = 593$, $z^* = 1.960$.

Conditions $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ are met. Thus, a 95% confidence interval for the percentage of all auto accidents that involve teenage drivers is given by

$0.14 \pm 1.960 \sqrt{\frac{0.14(1-0.14)}{593}} \approx 0.14 \pm 0.028 = (0.112, 0.168)$, hence in percents it will be $14\% \pm 2.8\% = (11.2\%, 16.8\%)$.

Answer: $14\% \pm 2.8\% = (11.2\%, 16.8\%)$.