Answer on Question #67463 – Math – Discrete Mathematics

Question

If A and B are the set of even integers and set of odd integers, respectively, find $A \cup B$ and $(A \cup B)^c$.

Solution

The set of even integers formally is

$$A = \{n|n = 2k, where\ k \in \mathbb{Z}\},$$

and the set of odd integers formally is

$$B = \{n|n = 2k + 1, where\ k \in \mathbb{Z}\}.$$

Their union is

$$A \cup B = \{n|n \in A\ or\ n \in B\} = \{n|n = 2k\ or\ n = 2k + 1, where\ k \in \mathbb{Z}\}.$$

Since every integer n is either even or odd, every integer n belongs to the union: $n \in A \cup B$, thus the union contains the set of all integers:

$$\mathbb{Z} \subseteq A \cup B.$$

On the other hand, both A and B are subsets of $\mathbb{Z}$, therefore their union is a subset of $\mathbb{Z}$ as well:

$$A \cup B \subseteq \mathbb{Z}.$$

These two inclusions imply the equality:

$$A \cup B = \mathbb{Z}$$

That is, the union of A and B is the set of all integers.

Then the complement of the union consists of non-integer numbers:

$$(A \cup B)^c = \{n|n \notin A \cup B\} = \{n|n \notin \mathbb{Z}\}.$$

Answer:

$A \cup B = \mathbb{Z}$ and $(A \cup B)^c = \{n|n \notin \mathbb{Z}\}$.