Answer on Question #67324 – Math – Algebra

Question

Find range of

$$f(x) = \frac{4}{6sinx + 5cosx + 7}$$

Solution

Rewrite

$$6sinx + 5cosx = |6^2 + 5^2 = 61| = \sqrt{61}\left(\frac{6}{\sqrt{61}} \cdot sinx + \frac{5}{\sqrt{61}} \cdot cosx\right)$$
$$= \left|\text{there exists } \varphi \text{ such that } cos\varphi = \frac{6}{\sqrt{61}}, \quad sin\varphi = \frac{5}{\sqrt{61}},\right.$$
$$\left. cos^2\varphi + sin^2\varphi = 1 \right| =$$

$= \sqrt{61}(cos\varphi \cdot sinx + sin\varphi \cdot cosx) = \sqrt{61}sin(x + \varphi)$.

Using the previous formula and inequalities

$$-1 \leq sin(x + \varphi) \leq 1$$

one gets

$$-\sqrt{61} \leq 6sinx + 5cosx \leq \sqrt{61},$$

hence

$$7 - \sqrt{61} \leq 6sinx + 5cosx + 7 \leq 7 + \sqrt{61}$$

and finally

$$\frac{4}{6sinx+5cosx+7} \leq \frac{4}{7-\sqrt{61}} \text{ or } \frac{4}{6sinx+5cosx+7} \geq \frac{4}{7+\sqrt{61}}.$$

Therefore, the range of $f(x) = \frac{4}{6sinx+5cosx+7}$ is

$$\left(-\infty, \frac{4}{7-\sqrt{61}}\right] \cup \left[\frac{4}{7+\sqrt{61}}, +\infty\right).$$

Answer: $\left(-\infty, \frac{4}{7-\sqrt{61}}\right] \cup \left[\frac{4}{7+\sqrt{61}}, +\infty\right)$.