$$\frac{dy}{dx} - ay = 0$$

$$\frac{dy}{y} = adx$$

$$\ln y = ax + lnC$$

$$y = C(x)e^{ax}$$

$$\frac{dy}{dx} = \frac{dC}{dx}e^{ax} + Cae^{ax}$$

a)

$$\frac{dC}{dx}e^{ax} + Cae^{ax} - Cae^{ax} = \mathrm{k}e^{\lambda(x)}$$

$$\frac{dC}{dx} = \frac{\mathrm{k}e^{\lambda(x)}}{e^{ax}}$$

$$C(x) = \int \frac{\mathrm{k}e^{\lambda(x)}}{e^{ax}}dx$$

Answer:

$$y = (\int \frac{\mathrm{k}e^{\lambda(x)}}{e^{ax}}dx) * e^{ax}$$

B)

$$\frac{dC}{dx}e^{ax} + Cae^{ax} - Cae^{ax} = \mathrm{k}e^{ax}$$

$$\frac{dC}{dx} = \frac{\mathrm{k}e^{ax}}{e^{ax}}$$

$$C(x) = k\int xdx = kx + c_1$$

Answer:

$$y = (kx + c_1) * e^{ax}$$

c)

$$\frac{dC}{dx}e^{ax} + Cae^{ax} - Cae^{ax} = \mathrm{k}x^n e^{ax}$$

$$\frac{dC}{dx} = \mathrm{k}x^n$$

$$C(x) = k\int x^n dx = k\frac{x^{n+1}}{n+1} + c_1$$

Answer:

$$y = (k\frac{x^{n+1}}{n+1} + c_1) * e^{ax}$$

12)

$$\frac{xdy}{dx} + y = y^2 lnx$$

$$z = \frac{1}{y}$$

$$dz = -y^{-2}dy$$

$$dy = -y^2 dz$$

$$-\frac{xy^2 dz}{dx} + y = y^2 lnx$$

$$-\frac{xdz}{dx} + \frac{1}{y} = lnx$$

$$-\frac{xdz}{dx} + z = lnx$$

$$-\frac{dz}{dx} + \frac{z}{x} = \frac{lnx}{x}$$

$$t = \frac{z}{x}$$

$$z = tx$$

$$dz = tdx + xdt$$

$$\frac{tdx + xdt}{dx} - t = \frac{lnx}{x}$$

$$t + \frac{xdt}{dx} - t = \frac{lnx}{x}$$

$$t = \int \frac{lnx}{x^2} dx = \left(u = lnx, dv = \frac{1}{x^2} dx\right) = uv - \int vdu = -\frac{1}{x} lnx + \int \frac{1}{x^2} = -\frac{1}{x} lnx - \frac{1}{x} + c_1$$

$$z = (-\frac{1}{x} lnx - \frac{1}{x} + c_1)x$$

Answer:

$$y = \frac{1}{z} = \frac{1}{(-\frac{1}{x} lnx - \frac{1}{x} + c_1)x}$$

13)

$$2cosx \frac{dy}{dx} - ysinx + \frac{1}{y} = 0$$

$$z = y^2$$

$$dz = 2ydy$$

$$\frac{2cosx}{2y}\frac{dz}{dx} - ysinx + \frac{1}{y} = 0$$

$$cosx\frac{dz}{dx} - y^2sinx + 1 = 0$$

$$cosx\frac{dz}{dx} - zsinx + 1 = 0$$

$$\frac{dz}{dx} - \frac{zsinx}{cosx} + \frac{1}{cosx} = 0$$

$$\frac{dz}{dx} - ztanx + \frac{1}{cosx} = 0$$

$$\frac{dz}{dx} - ztanx = 0$$

$$\frac{dz}{dx} = ztanx$$

$$\frac{dz}{z} = tanxdx$$

$$lnz = -\ln(cosx) + lnc$$

$$z_1 = -cosx + c$$

$$z_2 = -\frac{A}{cosx}$$

$$\frac{dz}{dx} = A\frac{tanx}{cosx}$$

$$A\frac{tanx}{cosx} - \left(-\frac{A}{cosx}\right)tanx = -\frac{1}{cosx}$$

$$2A\frac{tanx}{cosx} = -\frac{1}{cosx}$$

$$2Atanx = -1$$

$$A = -\frac{1}{tanx}$$

$$z_2 = \frac{1}{tanx}\frac{1}{cosx}$$

Answer:

$$z = z_1 + z_2 = \frac{1}{tanx}\frac{1}{cosx} - cosx + c$$

14)

$$\frac{dy}{dx} = (x + y + 1)(x + y - 1)$$

$$u = x + y$$

$$du = dx + dy$$

$$dy = du - dx$$

$$\frac{du - dx}{dx} = (u + 1)(u - 1)$$

$$\frac{du}{dx} - 1 = (u + 1)(u - 1)$$

$\frac{du}{dx} = u^2$-1+1

$$\frac{du}{dx} = u^2$$

$$\frac{du}{u^2} = dx$$

$$\int \frac{du}{u^2} = \int dx$$

$$x = -\frac{1}{u} + c$$

$$u = \frac{1}{c - x}$$

$$\frac{1}{c - x} = x + y$$

Answer:

$$y = x - \frac{1}{c - x}$$

15)

$$\frac{dy}{dx} = (y - x - 2)^2$$

$$u = y - x - 2$$

$$du = dy - dx$$

$$dy = du + dx$$

$$\frac{du + dx}{dx} = (u)^2$$

$$\frac{du}{dx} = (u)^2 - 1$$

$$\int \frac{du}{(u)^2 - 1} = \int dx$$

$$x = \frac{1}{2}\left(\frac{u-1}{u+1}\right) + c$$

$$u = \frac{1 + e^{2(x+c)}}{1 - e^{2(x+c)}}$$

$$y - x - 2 = \frac{1 + e^{2(x+c)}}{1 - e^{2(x+c)}}$$

Answer:

$$y = \frac{1 + e^{2(x+c)}}{1 - e^{2(x+c)}} + x + 2$$