Answer on Question #67231 - Math – Analytic Geometry

Question: Find the equations of the straight lines which pass through the intersection of $3x - 4y + 1 = 0$ and $5x + y = 1$ and which cut off equal intercepts from the axes.

Solution: First, let's find the intersection of $3x - 4y + 1 = 0$ and $5x + y = 1$.

We have to solve the system of equations:

$$\begin{cases} 3x - 4y + 1 = 0, \\ 5x + y = 1. \end{cases}$$

We multiply the second equation of the system by 4 :

$$\begin{cases} 3x - 4y = -1, \\ 20x + 4y = 4, \end{cases}$$

and add two equations in order to find x:

$$\begin{cases} 23x = 3, \\ y = 1 - 5x; \end{cases}$$

The solution of the system is

$$\begin{cases} x = \dfrac{3}{23}, \\ y = \dfrac{8}{23}. \end{cases}$$

Now, we have to find the equations of the straight lines which pass through the point $P\left(\frac{3}{23}, \frac{8}{23}\right)$ and which cut off equal intercepts from the axes.

Equation of the straight line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$. Given that intercepts are equal. Thus, $a = b$. Since the line passes through $P\left(\frac{3}{23}, \frac{8}{23}\right)$

$$\frac{3}{23a} + \frac{8}{23a} = 1.$$

Hence $a = \frac{3}{23} + \frac{8}{23} = \frac{11}{23}$ and the line is unique with the equation $23x + 23y = 11$.

Answer: There exists unique line, its equation is $23x + 23y = 11$.