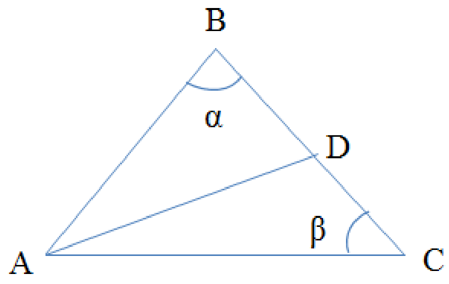
Answer on Question #67211 - Math - Analytic Geometry

Question

Prove that if ABC is triangle and Dis the mid point of BC , then AB^2+AC^2,=2*(AD^2+DC^2)

Solution

Draw in triangle ABC segment AD. Since D is the midpoint of BC, so AD is the median from A to BC Consider triangle ABD. Use the Law of cosines (or cosine rule). We have



$$AD^2 = AB^2 + BD^2 - 2AB \cdot BD \cdot \cos\alpha \qquad (1)$$

Consider triangle ACD. Use the Law of cosines. We have

$$AD^2 = AC^2 + CD^2 - 2AC \cdot CD \cdot \cos\beta \qquad (2)$$

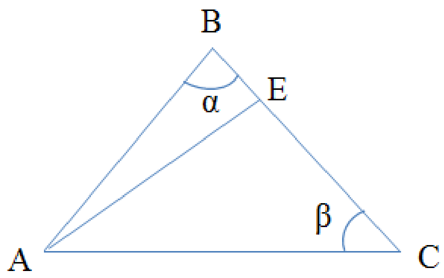
Add (1) and (2):

$$2AD^2 = AB^2 + BD^2 + AC^2 + CD^2 - 2AB \cdot BD \cdot \cos\alpha - 2AC \cdot CD \cdot \cos\beta$$

Take into account that $BD = DC$. Then we get

$$2AD^2 = AB^2 + AC^2 + 2DC^2 - 2DC(AB \cdot \cos\alpha + AC \cdot \cos\beta) \qquad (3)$$

Now we draw in triangle ABC the altitude AE. We get two right triangles ABE and ACE



In triangle ABE we have

$$BE = AB \cdot \cos\alpha \qquad (4)$$

and in triangle ACE we have

$$CE = AC \cdot \cos\beta \qquad (5)$$

Substituting (4) and (5) into (3) we get

$$2AD^2 = AB^2 + AC^2 + 2DC^2 - 2DC(BE + CE) \qquad (6)$$

however$BE + CE = BC = 2DC$. Then for (6) we get

$$2AD^2 = AB^2 + AC^2 - 2DC^2$$

And finally

$$AB^2 + AC^2 = 2(AD^2 + DC^2)$$

Answer: if ABC is triangle and D is the mid point of BC, then

$$AB^2 + AC^2 = 2(AD^2 + DC^2)$$