Question #6736 .A woman is on a lake in a canoe 1km from the closest point P of a straight shoreline. She wants to get to a point Q, 10km along the shore from P. To do so, she paddles to a point R between P and Q and then walks the remaining distance to Q. She can paddle 3km/hour and walk 5km/hour. How should she pick the point R so that she gets to Q as quickly as possible?

Solution. Suppose that now she is at the point X. then $PX = 1$, suppose that the desirable point $R \in [P, Q]$ and $RP = x$. Due to Pythagorean theorem $XR = \sqrt{1+x^2}$. The time womand spends on paddling from X to R and walking from R to Q(that is $10-x$ km):

$$f(x) := \frac{\sqrt{1+x^2}}{3} + \frac{10-x}{5}$$

Minimizing the last with respect to x. Take derivative $f'(x) = \frac{x}{3\sqrt{1+x^2}} - 1/5 = 0$, hence $x = 3/4$. On this plan she will spend $34/15$ h. We also must chek points $x = 0$ and $x = 10$(this obviously does not interest us, as t will certainly give bigger time). $f(0) = 1/3 + 10/5 = 35/15 > 34/15$.

Answer $PR = 3/4$.