Answer on Question #67156 – Math – Linear Algebra

Question

Under what condition A.B is not equal to zero and A×B is equal to zero when A and B are two non-zero vectors?

Solution

$\mathbf{A}.\mathbf{B}$ is not equal to zero and $\mathbf{A}\times\mathbf{B}$ is equal to zero when vectors $\mathbf{A}$ and $\mathbf{B}$ are collinear.

Let's show it.

Let $\mathbf{B}$ be $\lambda\mathbf{A}$, that is,

$$\mathbf{B} = (B_x, B_y, B_z) = (\lambda A_x, \lambda A_y, \lambda A_z),$$

then

$$\mathbf{A}.\mathbf{B} = A_xB_x + A_yB_y + A_zB_z = \lambda(A_xA_x + A_yA_y + A_zA_z) = \lambda(A_x^2 + A_y^2 + A_z^2).$$

If $\mathbf{A}$ is a non-zero vector, then $\mathbf{A}.\mathbf{B} = \lambda(A_x^2 + A_y^2 + A_z^2) > 0$.

Next, we find $\mathbf{A}\times\mathbf{B}$:

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ \lambda A_x & \lambda A_y & \lambda A_z \end{vmatrix}$$

$$= \mathbf{i}\begin{vmatrix} A_y & A_z \\ \lambda A_y & \lambda A_z \end{vmatrix} - \mathbf{j}\begin{vmatrix} A_x & A_z \\ \lambda A_x & \lambda A_z \end{vmatrix} + \mathbf{k}\begin{vmatrix} A_x & A_y \\ \lambda A_x & \lambda A_y \end{vmatrix} =$$

$$= \mathbf{i}(\lambda A_yA_z - \lambda A_yA_z) - \mathbf{j}(\lambda A_xA_z - \lambda A_xA_z) + \mathbf{k}(\lambda A_xA_y - \lambda A_xA_y) = 0$$

Answer: if vectors $\mathbf{A}$ and $\mathbf{B}$ are collinear, non-zero vectors, then $\mathbf{A}.\mathbf{B} \neq 0$ and $\mathbf{A}\times\mathbf{B} = 0$.