# Answer on Question #67151 – Math – Differential Equations

## Question

A chain hangs over a nail with 2.0 m on one side and 6.0 m on the other side. If the force of friction is equal to the weight of 1.0 m of the chain, calculate the time required for the chain to slide off the nail.

## Solution

Let $l = 8$ be the length of the chain, $x(t)$ be the length of the longest side of the chain, $m$ be the mass of one meter of the chain.
Mass of the chain is $M = ml$, the friction force is $m \cdot 1 \cdot g = mg$.

$F = Ma$.

$mx(t)g - m(l - x(t))g - mg = Ma$.

$2mgx(t)g - mlg - mg = ml\frac{d^2x}{dt^2}$.

$$\frac{d^2x}{dt^2} - 2\frac{g}{l}x(t) = -\frac{(l+1)g}{l}. \quad \textbf{(1)}$$

Let $x(t) = C$ be a particular solution of nonhomogeneous differential equation (1), where $C$ is a real constant.
Then

$\frac{d^2C}{dt^2} - 2\frac{g}{l}C = -\frac{(l+1)g}{l}$,

$-2\frac{g}{l}C = -\frac{(l+1)g}{l}$,

hence $C = \frac{l+1}{2}$.

The general solution of the homogeneous equation $\frac{d^2x}{dt^2} - 2\frac{g}{l}x(t) = 0$ is

$x_*(t) = Ae^{\sqrt{\frac{2g}{l}}t} + Be^{-\sqrt{\frac{2g}{l}}t}$.

The general solution of nonhomogeneous differential equation (1) is

$x(t) = Ae^{\sqrt{\frac{2g}{l}}t} + Be^{-\sqrt{\frac{2g}{l}}t} + \frac{l+1}{2}$;

$x(t) = Ae^{1.566t} + Be^{-1.566t} + 4.5$.

Initial conditions: $x(0) = 6, \frac{dx}{dt} = 0$.

So $\begin{cases} A + B + 4.5 = 6 \\ A - B = 0 \end{cases} \rightarrow \quad A = B = 0.75.$

$x(t) = 0.75e^{1.566t} + 0.75e^{-1.566t} + 4.5.$

When $x(t) = 8$:
$0.75e^{1.566t} + 0.75e^{-1.566t} + 4.5 = 8 \ \rightarrow e^{1.566t} + e^{-1.566t} - 4.667 = 0 \rightarrow$
$e^{2*1.566t} - 4.667e^{1.566t} + 1 = 0 \ \ \rightarrow \ \ e^{1.566t} = 0.225 \ \ or \ \ e^{1.566t} = 4.442.$

If $e^{1.566t} = 0.225$ then $t = -0.953 < 0$ is impossible.

If $e^{1.566t} = 4.442$ then $t = 0.953\ sec.$
**Answer:** $t = 0.953\ sec.$