Answer on Question #67101 – Math – Algebra

Question

(true/false)with justification.
There is no real number 'a' such that the equation
x^2 + ax - 3 = 0 has equal roots

Solution

$x^2 + ax - 3 = 0,$
$D = b^2 - 4ac = a^2 + 3 * 4 = a^2 + 12,$

$x_1 = \frac{-a+\sqrt{D}}{2} = \frac{-a+\sqrt{a^2+12}}{2},$

$x_2 = \frac{-a-\sqrt{D}}{2} = \frac{-a-\sqrt{a^2+12}}{2}.$

If the equation has equal roots, then a can be found from the following equation:

$x_1 = x_2,$

$\frac{-a+\sqrt{a^2+12}}{2} = \frac{-a-\sqrt{a^2+12}}{2},$

$2(-a + \sqrt{a^2 + 12}) = 2(-a - \sqrt{a^2 + 12}),$

$-2a + 2a = 2(-\sqrt{a^2 + 12} - \sqrt{a^2 + 12}),$

$-4\sqrt{a^2 + 12} = 0.$

It should be noted that

$\sqrt{a^2 + 12} \geq \sqrt{12} > 0$, so $\sqrt{a^2 + 12} \neq 0$.
Thus, there is no real number 'a' such that the equation
x^2 + ax - 3 = 0 has equal roots.
It is True.

Answer: True.