Question

Solve the equation

$$(x + 1)(x + 3)(x + 4)(x + 6) = 112$$

Solution

$(x + 1)(x + 3)(x + 4)(x + 6) = 112,$

$[(x + 1)(x + 3)] \cdot [(x + 4)(x + 6)] = 112,$

$(x^2 + 4x + 3)(x^2 + 10x + 24) = 112,$

$x^4 + 10x^3 + 24x^2 + 4x^3 + 40x^2 + 96x + 3x^2 + 30x + 72 = 112,$

$x^4 + 14x^3 + 67x^2 + 126x - 40 = 0,$

$x^2(x^2 + 7x - 2) + 7x(x^2 + 7x - 2) + 20(x^2 + 7x - 2) = 0,$

$(x^2 + 7x - 2)(x^2 + 7x + 20) = 0,$

$(x^2 + 7x - 2) = 0$ or $(x^2 + 7x + 20) = 0.$

If

$x^2 + 7x - 2 = 0,$

then

$D = 7^2 - 4 \cdot 1 \cdot (-2) = 49 + 8 = 57,$

the roots of the equation are

$x_1 = \dfrac{-7 + \sqrt{57}}{2};$

$x_2 = \dfrac{-7 - \sqrt{57}}{2}.$

If

$x^2 + 7x + 20 = 0,$

then

$D = 49 - 80 = -31,$

$$\sqrt{D} = \pm\sqrt{31}i,$$

the roots of the equation are

$$x_1 = \frac{-7 + \sqrt{31}i}{2},$$

$$x_2 = \frac{-7 - \sqrt{31}i}{2}.$$

Thus, the roots of the initial equation $(x + 1)(x + 3)(x + 4)(x + 6) = 112$ are

$$\frac{-7+\sqrt{57}}{2}, \frac{-7-\sqrt{57}}{2}, \frac{-7+\sqrt{31}i}{2}, \frac{-7-\sqrt{31}i}{2}.$$

Answer: $\frac{-7+\sqrt{57}}{2}, \frac{-7-\sqrt{57}}{2}, \frac{-7+\sqrt{31}i}{2}, \frac{-7-\sqrt{31}i}{2}.$