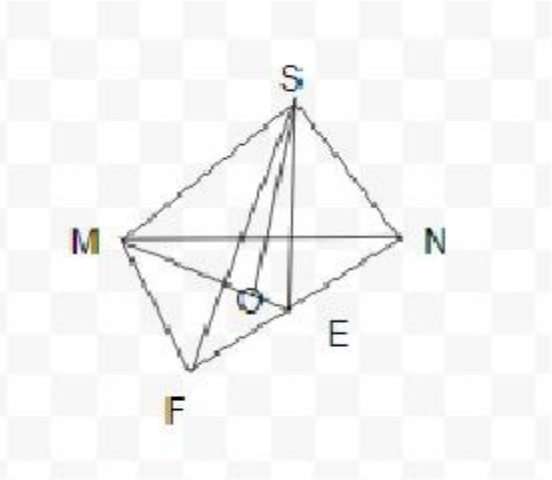
ANSWER ON QUESTION #66883 – MATH – GEOMETRY

QUESTION

SMNF is a regular triangular pyramid. SO (height)=6cm. Measure of the SEO angle is 60 degrees($\angle SEO = 60°$). Find: MF, apothem SE, total area of pyramid, volume of pyramid and the area of the SME triangle.



SOLUTION

1) Since $SMNF$ is a regular triangular pyramid, the base of the height falls in the centroid of the triangle $\Delta\, MNF$.

2) Since $SMNF$ is a regular triangular pyramid, the triangle $\Delta\, MNF$ is regular. This means that ME is a height, median, bisector.

$$\left.\begin{matrix} SO \perp ME \\ ME \perp NF \end{matrix}\right\} \rightarrow SE \perp NF \text{ by the theorem of three perpendiculars.}$$

Conclusion, SE is an apothem.

3) Consider a triangle ΔSEO:

$SO \perp OE$, hence the triangle ΔSEO is right.

$$\left.\begin{matrix} \angle SEO = 60° \\ SO = 6 \end{matrix}\right\} \rightarrow \tan\angle SEO = \frac{SO}{OE} \rightarrow OE = \frac{SO}{\tan\angle SEO} = \frac{SO}{\tan 60°} = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3};$$

$$\sin\angle SEO = \frac{SO}{SE} \rightarrow SE = \frac{SO}{\sin\angle SEO} = \frac{SO}{\sin 60°} = \frac{6}{\frac{\sqrt{3}}{2}} = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}.$$

$$\boxed{\begin{array}{c} SE = 4\sqrt{3}\ cm \text{ is an apothem} \\ OE = 2\sqrt{3}\ cm \end{array}}$$

4) Consider a regular triangle ΔMNF.

O is the center of mass of a triangle. As we know, the center of mass divides the median in the ratio 2 to 1 counting from the top of the triangle.
In this case,

$$\frac{MO}{OE} = \frac{2}{1} \to MO = 2OE \to ME = MO + OE = 2OE + OE = 3OE = 3 \cdot 2\sqrt{3} = 6\sqrt{3}$$

$$\boxed{ME = 6\sqrt{3}\ cm \text{ is a height of a regular triangle}}$$

We write down the formula for the height of a regular triangle

$$h = \frac{a\sqrt{3}}{2}, a \text{ is the length of triangle side}$$

In this case,

$$ME = \frac{MF\sqrt{3}}{2} \to MF = \frac{2ME}{\sqrt{3}} = \frac{2 \cdot 6\sqrt{3}}{\sqrt{3}} = 12$$

$$\boxed{MF = 12\ cm \text{ is a side of the regular pyramid}}$$

The area of the base is

$$A_1 = A_{\Delta MNF} = \frac{MF^2\sqrt{3}}{4} = \frac{12^2\sqrt{3}}{4} = 36\sqrt{3}$$

By the definition, the volume of the pyramid is

$$V = \frac{1}{3}S_1 \cdot h = \frac{1}{3} \cdot 36\sqrt{3} \cdot 6 = 72\sqrt{3}$$

$$\boxed{V = 72\sqrt{3}\ cm^3}$$

By the definition,

$$A_{\Delta SME} = \frac{1}{2} \cdot SO \cdot ME = \frac{1}{2} \cdot 6 \cdot 6\sqrt{3} = 18\sqrt{3}\ cm^2$$

$$\boxed{A_{\Delta SME} = 18\sqrt{3}\ cm^2}$$

By the definition, the total area is

$$A_{total} = A_{base} + \frac{1}{2} \cdot P \cdot L,$$

where

$A_{base} = A_1$ is the area of the base,

P is the base perimeter,

L is an apothem.

In this case,

$$A_{total} = 36\sqrt{3} + \frac{1}{2} \cdot 3MF \cdot SE = 36\sqrt{3} + \frac{1}{2} \cdot 3 \cdot 12 \cdot 4\sqrt{3} =$$

$$= 54\sqrt{3} + 72\sqrt{3} = 126\sqrt{3}$$

$$\boxed{A_{total} = 126\sqrt{3} cm^2}$$

ANSWER:

$$MF = 12\ cm$$

$$SE = 4\sqrt{3}\ cm$$

$$A_{total} = 126\sqrt{3}\ cm^2$$

$$V = 72\sqrt{3}\ cm^3$$

$$A_{\Delta SME} = 18\sqrt{3}\ cm^2$$