Answer on Question #66867 – Math – Calculus

Question

Evaluate $\lim\limits_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$

Solution

In order to evaluate this limit we use the following properties of limits:

$$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a}f(x) + \lim_{x\to a}g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim\limits_{x\to a}f(x)}{\lim\limits_{x\to a}g(x)}$$

$$\lim_{x\to a}x = a$$

$$\lim_{x\to a}c = c$$

$$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a}f(x)}$$

We can't apply the Quotient Law immediately, since the limit of the numerator and

denominator are 0. In order to rationalize the numerator and denominator we need multiply them by conjugate:

$$\lim_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}} = \lim_{x\to 3}\frac{\left(\sqrt{3x}-3\right)\left(\sqrt{3x}+3\right)}{\left(\sqrt{2x-4}-\sqrt{2}\right)\left(\sqrt{2x-4}+\sqrt{2}\right)}\cdot\frac{\left(\sqrt{2x-4}+\sqrt{2}\right)}{\left(\sqrt{3x}+3\right)} =$$

$$= \lim_{x\to 3}\frac{3x-9}{2x-4-2}\cdot\frac{\left(\sqrt{2x-4}+\sqrt{2}\right)}{\left(\sqrt{3x}+3\right)} = \lim_{x\to 3}\frac{3(x-3)}{2(x-3)}\cdot\frac{\left(\sqrt{2x-4}+\sqrt{2}\right)}{\left(\sqrt{3x}+3\right)} =$$

$$= \frac{3}{2}\lim_{x\to 3}\frac{\left(\sqrt{2x-4}+\sqrt{2}\right)}{\left(\sqrt{3x}+3\right)} = \frac{3}{2}\frac{\left(\sqrt{\lim\limits_{x\to 3}(2x-4)}+\sqrt{2}\right)}{\left(\sqrt{\lim\limits_{x\to 3}3x}+3\right)} =$$

$$= \frac{3}{2}\frac{\left(\sqrt{2\cdot 3-4}+\sqrt{2}\right)}{\left(\sqrt{3\cdot 3}+3\right)} = \frac{3}{2}\cdot\frac{2\sqrt{2}}{6} = \frac{1}{\sqrt{2}}$$

Answer: $\lim\limits_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}} = \frac{1}{\sqrt{2}}$