Answer on Question # 66865- Math - Calculus

Question: Determine the interval on which the given function is continuous

$$f(x) = \begin{cases} \sin\frac{1}{x}, & if\ \ x \neq 0; \\ 0, & if\ x = 0. \end{cases}$$

Solution: The function $h(x) = \frac{1}{x}$ is continuous on $(-\infty, 0) \cup (0, +\infty)$. The function $g(x) = \sin x$ is continuous for arbitrary real x. Therefore, $f(x) = \sin\frac{1}{x} = g(h(x))$ is continuous on $(-\infty, 0) \cup (0, +\infty)$ as a composition of continuous functions.

Let us now investigate the point $x = 0$. The $\lim\limits_{x\to 0} f(x)$ does not exist. Indeed, let us take two sequences $x_n = \frac{1}{2\pi n}$ and $x'_n = \frac{1}{\frac{\pi}{2}+2\pi n}$, $n \in N$. Both of them converge to 0 as $n \to +\infty$, but $\lim\limits_{n\to+\infty} f(x_n) = \lim\limits_{n\to+\infty} \sin 2\pi n = 0$ and $\lim\limits_{n\to+\infty} f(x'_n) = \lim\limits_{n\to+\infty} \sin\left(\frac{\pi}{2} + 2\pi n\right) = 1$. Since $0 \neq 1$, the $\lim\limits_{x\to 0} f(x)$ does not exist. Therefore, $x = 0$ is a point of discontinuity for the function $f(x)$.

The function $f(x)$ is continuous on $(-\infty, 0)$ and on $(0, +\infty)$, or on the arbitrary subinterval of the mentioned intervals.

Answer: The function $f(x)$ is continuous on the set $(-\infty, 0) \cup (0, +\infty)$.