Answer on Question #66776 – Math – Statistics and Probability

Question

If the second moment of a Poisson distribution is 6, find the probability $P(X \geq 2)$.

Solution

If X has a Poisson distribution, then

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}, \lambda > 0, k = 0, 1, 2, \dots$$

(see https://en.wikipedia.org/wiki/Poisson_distribution#Definition).

From the definition of the second moment (see https://en.wikipedia.org/wiki/Moment_(mathematics)) we get

$$\mathbb{E}(X^2) = 6.$$

On the other hand,

$$\mathbb{E}(X^2) = Var(X) + [\mathbb{E}(X)]^2$$

(see https://en.wikipedia.org/wiki/Variance#Definition).

Since X has a Poisson distribution then

$\mathbb{E}(X) = Var(X) = \lambda$

(see https://en.wikipedia.org/wiki/Poisson_distribution#Definition).

We have

$6 = \lambda + \lambda^2 \Rightarrow \lambda^2 + \lambda - 6 = 0.$

Let us solve the latter equation. Using the quadratic formula (see https://en.wikipedia.org/wiki/Quadratic_formula) we obtain

$$\lambda = \frac{-1 \pm \sqrt{1+24}}{2} \Rightarrow \left[\begin{matrix} \lambda_1 = 2 \\ \lambda_2 = -3 \end{matrix} \right..$$

Since $\lambda_2 = -3 < 0$ we conclude that $\lambda = 2$ and

$$P(X = k) = \frac{2^k}{k!} e^{-2}.$$

Then required probability is

$P(X \geq 2) = 1 - P(X < 2) =$

$= 1 - [P(X = 0) + P(X = 1)] = 1 - [e^{-2} + 2e^{-2}] = 1 - 3e^{-2} = 1 - \frac{3}{e^2}.$

Answer: $1 - \frac{3}{e^2}.$