Answer on Question #66760 – Math – Calculus

Question

Obtain the directional derivative for a scalar field at the point.

Solution

The rate of change of $f(x,y)$ in the direction of the unit vector $\vec{u} = \{a,b\}$ is called the directional derivate and is denoted by $D_{\vec{u}} f(x,y)$. The definition of the directional derivate is

$$D_{\vec{u}} f(x,y) = \lim_{h\to 0} \frac{f(x+ah, y+bh) - f(x,y)}{h}.$$

So, the definition of the directional derivative is very similar to the definition of partial derivatives. However, in practice this can be a very difficult limit to compute so we need an easier way of taking directional derivatives. It's actually fairly simple to derive an equivalent formula for taking directional derivatives.

$$D_{\vec{u}} f(x,y) = f_x(x,y)a + f_y(x,y)b.$$

Or, if we want to use the standard basis vectors, the gradient is

$$\nabla f = f_x \vec{i} + f_y \vec{j}.$$

The definition is only shown for functions of two or three variables, however there is a natural extension to functions of any number of variables that we'd like.

With the definition of the gradient we can now say that the directional derivative is given by

$$D_{\vec{u}} f = \nabla f \cdot \vec{u},$$

where we will no longer show the variable and use this formula for any number of variables.