

## Answer on Question #66744 – Math – Differential Equations

### Question

A box is to have square base an open top and volume of 32 meter cube. Find the dimension of the box that box that uses the least amount of material.

### Solution

$$V = x^2 h = 32 \rightarrow h = \frac{32}{x^2}.$$

$$S = x^2 + 4xh = x^2 + \frac{128}{x}.$$

$$\frac{dS}{dx} = 0 \rightarrow 2x - \frac{128}{x^2} = 0 \rightarrow x = \sqrt[3]{64} = 4.$$

$$\frac{d^2S}{dx^2}(4) = \frac{256}{x^3} \Big|_{x=4} = \frac{256}{4^3} > 0.$$

So  $S$  has minimum at  $x_* = 4$  and

$$h_* = \frac{32}{x_*^2} = \frac{32}{4^2} = \frac{32}{16} = 2.$$

The box uses the least amount of material when  $x = 4 \text{ m}$ ,  $h = 2 \text{ m}$ .

**Answer:** 4 m, 4 m, 2 m.