Answer on Question #66711 – Math – Complex Analysis

Question

Find all the 8th roots of i3 − 3. Also show any one of them in an Argand diagram.

Solution

$$z = -3 + 3i = 3\sqrt{2}\left(cos\frac{3\pi}{4} + isin\frac{3\pi}{4}\right)$$

$$\sqrt[8]{z} = \sqrt[8]{3\sqrt{2}}\left(cos\frac{3\pi+8\pi n}{32} + isin\frac{3\pi+8\pi n}{32}\right),\ n = 0,1,\dots,7.$$

$$z_1 = \sqrt[16]{18}\left(cos\frac{3\pi}{32} + isin\frac{3\pi}{32}\right);\quad z_2 = \sqrt[16]{18}\left(cos\frac{11\pi}{32} + isin\frac{11\pi}{32}\right);$$
$$z_3 = \sqrt[16]{18}\left(cos\frac{19\pi}{32} + isin\frac{19\pi}{32}\right);\ z_4 = \sqrt[16]{18}\left(cos\frac{27\pi}{32} + isin\frac{27\pi}{32}\right);$$
$$z_5 = \sqrt[16]{18}\left(cos\frac{35\pi}{32} + isin\frac{35\pi}{32}\right);\ z_6 = \sqrt[16]{18}\left(cos\frac{43\pi}{32} + isin\frac{43\pi}{32}\right);$$
$$z_7 = \sqrt[16]{18}\left(cos\frac{51\pi}{32} + isin\frac{51\pi}{32}\right);\ z_8 = \sqrt[16]{18}\left(cos\frac{59\pi}{32} + isin\frac{59\pi}{32}\right);$$

Argand diagram for $z_3 = \sqrt[16]{18}\left(cos\frac{19\pi}{32} + isin\frac{19\pi}{32}\right)$