Answer on Question #66700 - Math – Calculus

Question

A cylindrical tank with radius 3 m is being filled with water at a rate of 4 m^3/min. How fast is the height of the water increasing?

Solution

Let R be the radius of the tank, $H(t)$ the height of the water at time$t$, and $V(t)$ the volume of the water. The quantities $V(t)$, R and $H(t)$ are related by the equation

$$V(t) = \pi R^2 H(t) \quad (1)$$

The rate of increase of the volume is the derivative with respect to time,

$$\frac{dV}{dt}$$

and the rate of increase of the height is

$$\frac{dH}{dt}$$

We can therefore restate the given and the unknown as follows

Given:

$$\frac{dV}{dt} = 4\text{m}^3/\text{min}$$

Unknown:

$$\frac{dH}{dt}$$

Now we take derivative of each side of (1) with respect to t:

$$\frac{dV}{dt} = \pi R^2 \frac{dH}{dt}$$

So

$$\frac{dH}{dt} = \frac{1}{\pi R^2}\frac{dV}{dt}$$

Substituting $R = 3$ m and $dV/dt = 4\text{m}^3/\text{min}$ we have

$$\frac{dH}{dt} = \frac{1}{\pi(3)^2} \cdot 4 = \frac{4}{9\pi}$$

Answer: the height of the water increasing at a rate of

$$\frac{dH}{dt} = \frac{4}{9\pi} \approx 0.14 \text{ m/min}$$