Answer on Question #66678 – Math – Linear Algebra

Question

Reduce the conic $x^2 - 6xy + y^2 - 4 = 0$ to standard form. Hence the given conic.

Solution

$$x^2 - 6xy + y^2 - 4 = 0$$

$$x = x' \cos\alpha - y' \sin\alpha$$

$$y = x' \sin\alpha + y' \cos\alpha$$

$$\tan 2\alpha = \frac{-6}{1-1} = \infty, \qquad 2\alpha = \frac{\pi}{2}, \qquad \alpha = \frac{\pi}{4}$$

$$x = \frac{\sqrt{2}}{2}(x' - y')$$

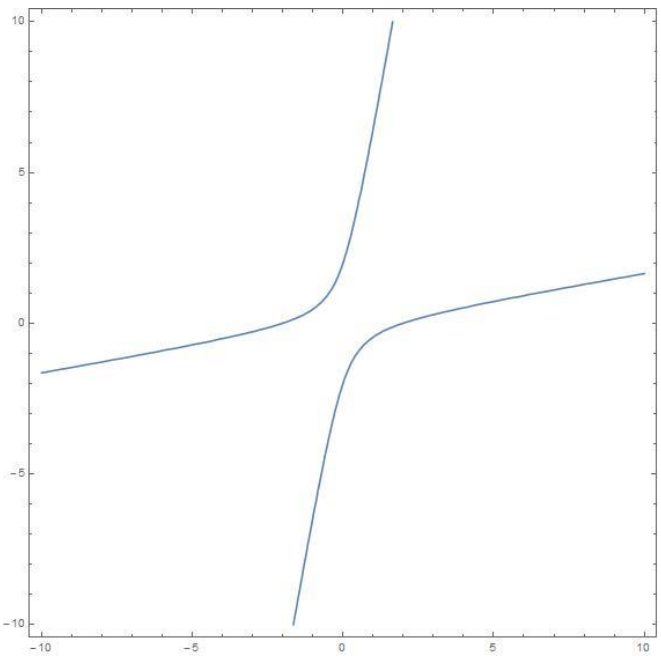
$$y = \frac{\sqrt{2}}{2}(x' + y')$$

$$\frac{1}{2}(x' - y')^2 - 6\times\frac{1}{2}(x' - y')(x' + y') + \frac{1}{2}(x' + y')^2 - 4 = 0$$

$$x'^2 + y'^2 - 2x'y' - 6x'^2 + 6y'^2 + x'^2 + y'^2 + 2x'y' - 8 = 0$$

$$-4x'^2 + 8y'^2 = 8$$

$$\frac{y'^2}{1} - \frac{x'^2}{2} = 1$$



Answer: The curve is a hyperbole.