Answer on Question #66677 – Math – Algorithms | Quantitative Methods

Question

determine a unique polynomial f(x) of degree<=3 such that
f(x_0)=1, f'(x_0)=2, f(x_1)=2, f'(x_1)=3 where x_1 - x_0 = h

Solution

$f(x) = ax^3 + bx^2 + cx + d.\ f'(x) = 3ax^2 + 2bx + c$

$$\begin{cases} f(x_0) = 1 \\ f(x_0 + h) = 2 \\ f'(x_0) = 2 \\ f'(x_0 + h) = 3 \end{cases} \rightarrow \begin{cases} ax_0^3 + bx_0^2 + cx_0 + d = 1 \\ a(x_0 + h)^3 + b(x_0 + h)^2 + c(x_0 + h) + d = 2 \\ 3ax_0^2 + 2bx_0 + c = 2 \\ 3a(x_0 + h)^2 + 2b(x_0 + h) + c = 3 \end{cases}$$

We can solve this system using Cramer's rule and obtain the coefficients a, b, c and d .

In particular case ($x_0 = 0$):

$$\begin{cases} d = 1 \\ ah^3 + bh^2 + ch + d = 2 \\ c = 2 \\ 3ah^2 + 2bh + c = 3 \end{cases} \rightarrow\ d = 1,\ c = 2 \rightarrow \begin{cases} ah^3 + bh^2 = 1 - 2h \\ 3ah^2 + 2bh = 1 \end{cases} \rightarrow$$

$$\rightarrow\ a = \frac{5h-2}{h^3},\ b = \frac{3-7h}{h^2}.$$

So $f(x) = \frac{5h-2}{h^3}x^3 + \frac{3-7h}{h^2}x^2 + 2x + 1.$