Answer on Question #66676 – Math – Statistics and Probability

Question

If the second moment of a Poisson distribution is 6, find the probability $P(X \geq 2)$.

Solution

We start from the definition of a Poisson distribution (see https://en.wikipedia.org/wiki/Poisson_distribution). We have

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}, k = 0, 1, \dots; \ \lambda > 0.$$

Further, we use the definition of moments (see https://en.wikipedia.org/wiki/Moment_(mathematics)). We have

$$E[X^2] = 6.$$

On the other hand, we know that

$$E[X] = \lambda; Var[X] = E[X^2] - (E[X])^2 = \lambda$$

(see https://en.wikipedia.org/wiki/Poisson_distribution). Then

$$E[X^2] = Var[X] + (E[X])^2 = \lambda + \lambda^2.$$

We have the following quadratic equation:

$$\lambda^2 + \lambda = 6 \Leftrightarrow \lambda^2 + \lambda - 6 = 0.$$

Using Vieta's formula (see https://brilliant.org/wiki/vietas-formula/) the roots are

$$\left[\begin{array}{l} \lambda = 2 \\ \lambda = -3 \end{array}\right..$$

Since λ must be positive we conclude that $\lambda = 2$, and X has the following distribution:

$$P(X = k) = \frac{2^k}{k!} e^{-2}.$$

Then

$$P(X \geq 2) = 1 - P(X < 2) = 1 - \big(P(X = 0) + P(X = 1)\big) =$$

$$= 1 - (e^{-2} + 2e^{-2}) = 1 - \frac{3}{e^2} = \frac{e^2 - 3}{e^2} \approx 0.594.$$

Answer: $\frac{e^2-3}{e^2}$.