

**Answer on Question #66664 – Math – Differential Equations
Question**

Solve the following DEs

i)

$$\left(\frac{dy}{dx} - 1\right)^2 \left(\frac{d^2y}{dx^2} + 1\right)^2 y = \left(\sin \frac{x}{2}\right)^2 + e^x + x$$

Solution

It is impossible to solve using analytical methods, and for numerical methods we have not initial value.

Question

ii)

$$2x^2y \frac{d^2y}{dx^2} + 4y^2 = x^2 \left(\frac{dy}{dx}\right)^2 + 2xy \frac{dy}{dx}$$

Solution

$$u = \frac{y'_x}{y}$$

$$u'_x + \frac{1}{2}u^2 - \frac{1}{x}u + \frac{2}{x^2} = 0$$

$$v(x) = e^{\frac{1}{2} \int u(x) dx}$$

$$x^2 v''_{xx} - x v'_x + v = 0$$

$$v(x) = |x|(C_1 + C_2 \ln|x|)$$

$$\ln v = \frac{1}{2} \int u(x) dx$$

$$\frac{v'_x}{v} = \frac{u}{2}$$

$$\frac{v'_x = C_1 + C_2 + C_2 \ln x}{C_1 + C_2 + C_2 \ln x} = \frac{y'_x}{2y}$$

$$\frac{C_1 + C_2 + C_2 \ln x}{x(C_1 + C_2 \ln x)} dx = \frac{dy}{2y}$$

$$\int \frac{C_1 + C_2 + C_2 \ln x}{x(C_1 + C_2 \ln x)} dx = \int \frac{dy}{2y}$$

Answer: $y = c_2 x^2 (c_1 + \ln x)^2$.