# Answer on Question # 66662 – Math – Differential Equations

## Question

Solve the following IVP
(d^2y)/(dx^2) + (dy)/(dx) - 2y = -6 sin2x -18 cos2x
y(0)=2, y'(0)=2

## Solution

We have IVP

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = -6\sin 2x - 18\cos 2x, \quad y(0) = 2, y'(0) = 2$$

Find first the general solution of this equation which can be written as

$$y = y_c + y_p$$

where $y_p$ is a particular solution of original equation and $y_c$ is the general solution of the related homogeneous equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

To find $y_c$ we substitute solution as exponential function $y = e^{mx}$ into the equation. We get

$$m^2 e^{mx} + m e^{mx} - 2e^{mx} = 0$$

or

$$(m^2 + m - 2)e^{mx} = 0$$

Then we get the auxiliary equation

$$m^2 + m - 2 = 0$$

with roots $m = 1$ and $m = -2$. So the solution of the complementary equation is

$$y_c = c_1 e^x + c_2 e^{-2x}$$

where $c_1$ and $c_2$ are some constants. Next we find a particular solution in the form

$$y_p = A\sin 2x + B\cos 2x$$

Then

$$y'_p = 2A\cos 2x - 2B\sin 2x$$
$$y''_p = -4A\sin 2x - 4B\cos 2x$$

Substituting $y_p$, $y'_p$ and $y''_p$ into the given differential equation gives

$$-4A\sin 2x - 4B\cos 2x + 2A\cos 2x - 2B\sin 2x - 2A\sin 2x - 2B\cos 2x = -6\sin 2x - 18\cos 2x$$

or

$$-(6A + 2B)\sin 2x - (6B - 2A)\cos 2x = -6\sin 2x - 18\cos 2x$$

This is true if

$$6A + 2B = 6 \quad \text{and} \quad 6B - 2A = 18$$

The solution of this system is $A = 0$ and $B = 3$. So, the particular solution is

$$y_p = 3\cos 2x$$

The general solution of the given equation is

$$y = c_1 e^x + c_2 e^{-2x} + 3\cos 2x$$

Now we find $c_1$ and $c_2$ using the initial conditions $y(0) = 2, y'(0) = 2$:

$$y(0) = c_1 e^0 + c_2 e^0 + 3\cos 0 = c_1 + c_2 + 3 = 2$$
$$y' = c_1 e^x - 2c_2 e^{-2x} - 6\sin 2x$$

$$y'(0) = c_1e^0 - 2c_2e^0 - 6\sin 0 = c_1 - 2c_2 = 2$$

We have system of equation

$$\begin{cases} c_1 + c_2 = -1 \\ c_1 - 2c_2 = 2 \end{cases}$$

The solution of this system is $c_1 = 0$, $c_2 = -1$, so the solution of IVP is

$$y = -e^{-2x} + 3\cos 2x$$

**Answer:** The solution of IVP is

$$y = -e^{-2x} + 3\cos 2x$$