Answer on Question #66476, Math / Calculus.

Use the mean value theorem to show that x is smaller than sin inverse x for x is greater than 0

Solution:

We will use the mean value theorem: if a function f is continuous on the closed interval $[a,b]$, and differentiable on the open interval (a,b), then there exists a point $c \in (a,b)$ such that

$$\frac{f(b)-f(a)}{b-a} = f'(c).$$

Consider the function $f(x)=\sin x$ and the closed interval $[0,x]$ for $x>0$. Since the function $f(x)=\sin x$ is differentiable, there is a point $c \in (0,x)$ such that

$$\frac{\sin x - \sin 0}{x-0} = \cos c.$$

Then

$$\frac{\sin x}{x} = \cos c.$$

Since $\cos x \le 1$,

$$\frac{\sin x}{x} \le 1.$$

Therefore $\sin x \le x$ for $x>0$. By the condition $\frac{\sin x}{x} = \cos c$ we have that $\sin = x$ when $\cos c = 1$. For $x>0$ and $c \in (0,x)$ we have that $\cos c = 1$ for $c \ge 2\pi$. Since $x > c$, $x > 2\pi$. But $\sin x \le 1$ for every x. Then by conditions $\sin x \le 1$ and $x > 2\pi$ we have that $\sin x < x$.

Hence $\sin x < x$ for $x > 0$.