Answer on Question #66371, Math, Calculus

True or false, why?

1) the level curves of $z = y/x$ are hyperbolas.

Solution.

The equation of level curves:

$$\frac{y}{x} = c \quad or \quad y = cx \ ; \ c - constant$$

It is not hyperbola.

Answer: false

2) $lim(x, y)$ tends to $(0,0)$

$$x^2 - \frac{y^2}{x^2} + y^2$$

does not exist.

Solution.

$$\lim_{(x,y)\to(0,0)} \left(x^2 - \frac{y^2}{x^2} + y^2 \right) = -\infty$$

Answer: true

3) the set

$$\{(x, y, z) | x > 0, y > 0, z < 0\}$$

is domain in R^3.

Solution.

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A function $f(x, y, z)$ can be defined on this set.

Answer: true

4) the work done by the force $F(x, y) = (-y, x)$ in moving a particle along the boundary of the ellipse

$$9x^2 + 4y^2 = 36$$

is 6.

Solution.

The work done by the Force $F(x, y)$:

$$W = \oint P(x, y)dx + Q(x, y)dy$$

We have:

$$P(x, y) = -y \ ; \ \ Q(x, y) = x$$

the equation of the ellipse:

$$\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$$

The parametric equation of the ellipse:

$$x = 2\cos t \ \ ; \ \ y = 3\sin t \ \ ; \ \ 0 \le y \le 2\pi$$

Then:

$$dx = -2\sin t\, dt \ \ ; \ \ dy = 3\cos t\, dt$$

$$P(x, y)dx + Q(x, y)dy = -ydx + xdy = -3\sin t\,(-2\sin t\, dt) + 2\cos t \cdot 3\cos t\, dt =$$

$$= 6(\sin t)^2 dt + 6(\cos t)^2 dt = 6dt$$

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$$W = 6\int_0^{2\pi} dt = 12\pi$$

Answer: false

5) the function $f(x, y, z) = e^x yz$ is integrable over $[0,1]\times[0,1]\times[0,1]$.

Solution.

The function $f(x, y, z)$ is defined over $[0,1]\times[0,1]\times[0,1]$, so it has integral :

$$\iiint e^x yz \,\mathrm{dxdydz} = \int_0^1 dx \int_0^1 dy \int_0^1 e^x yz dz = \int_0^1 dx \int_0^1 \frac{e^x y}{2} dy = \int_0^1 \frac{e^x}{4} dx = \frac{e}{4} - \frac{1}{4}$$

Answer: true