Answer on Question #66339 – Math – Differential Equations

Question

Find the equation of the integral surface of the differential equation

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$

which passes through the line $x = 1, y = 0$.

Solution

The equation of characteristics:

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$$

Then

$$\dot{x} - \dot{y} = x^2 - y^2 + z(x - y) = (x - y)(x + y + z);$$

$$\dot{z} - \dot{x} = z^2 - x^2 + y(z - x) = (z - x)(x + y + z);$$

$$\dot{z} - \dot{y} = z^2 - y^2 + x(z - y) = (z - y)(x + y + z);$$

$$\frac{d(x-y)}{d(z-x)} = \frac{x-y}{z-x};$$

$$\frac{d(x-y)}{x-y} = \frac{d(z-x)}{z-x};$$

$$\ln|x - y| = \ln|z - x| + \ln c_1;$$

$$c_1 = \frac{x-y}{z-x}.$$

In the same manner

$$\frac{d(x-y)}{d(z-y)} = \frac{x-y}{z-y};$$

$$\frac{d(x-y)}{x-y} = \frac{d(z-y)}{z-y};$$

$$\ln|x-y| = \ln|z-y| + \ln c_2;$$

$$c_2 = \frac{x-y}{z-y};$$

$$\frac{d(z-x)}{d(z-y)} = \frac{z-x}{z-y};$$

$$\frac{d(z-x)}{z-x} = \frac{d(z-y)}{z-y};$$

$$\ln|z-x| = \ln|z-y| + \ln c_3;$$

$$c_3 = \frac{z-x}{z-y}.$$

For $x = 1, y = 0$:

$$c_1 = \frac{1}{z-1} \quad ; \quad c_2 = \frac{1}{z} \quad ; \quad c_3 = \frac{z-1}{z}.$$

There are infinitely many solutions

$$z = c; \quad c \text{ is an arbitrary real constant.}$$

Answer: $z = c;$ c is an arbitrary real constant.