Answer on Question #66309 - Math - Linear Algebra

Question 66309:

An electronics company produces transistors, resistors, and computer chips. Each transistor requires 3 units of copper, 1 unit of zinc, and 2 units of glass. Each resistor requires 3, 2, and 1 units of the three materials, and each computer chip requires 2, 1, and 2 units of these materials, respectively. How many of each product can be made with 1570 units of copper, 740 units of zinc, and 880 units of glass? Solve this exercise by using the inverse of the coefficient matrix to solve a system of equations.

Solution:

Let t denotes the quantity of transistors, r denotes the quantity of resistors, and c denotes the quantity of computer chips. The following three linear equations correspond with this exercise:

$$\begin{cases} 3*t+3*r+2*c=1570 \\ 1*t+2*r+1*c=740 \\ 2*t+1*r+2*c=880 \end{cases}$$

The matrix form of this system is $Ax=b$, where A is the coefficient matrix, x is the variable vector, and b is the constant vector:

$$A=\begin{pmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}, \quad x=\begin{pmatrix} t \\ r \\ c \end{pmatrix}, \quad b=\begin{pmatrix} 1570 \\ 740 \\ 880 \end{pmatrix}.$$

If the inverse of the coefficient matrix, A^{-1}, exists, then the solution is $x=A^{-1}b$.

The inverse matrix A^{-1} can be found by using the following formula:

$$A^{-1}=\frac{1}{\det A}(C_A)^T,$$

where $\det A$ is the determinant of A, C_A is the matrix of cofactors of A.
Now, apply this formula to find A^{-1}:

$$\det A=\begin{vmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}=3*(4-1)-3*(2-2)+2*(1-4)=9-6=3.$$

(Note that $\det A\neq 0$, thus the matrix is invertible.)

$$C_A=\begin{pmatrix} 4-1 & -(2-2) & 1-4 \\ -(6-2) & 6-4 & -(3-6) \\ 3-4 & -(3-2) & 6-3 \end{pmatrix}=\begin{pmatrix} 3 & 0 & -3 \\ -4 & 2 & 3 \\ -1 & -1 & 3 \end{pmatrix}.$$

$$(C_A)^T=\begin{pmatrix} 3 & 0 & -3 \\ -4 & 2 & 3 \\ -1 & -1 & 3 \end{pmatrix}^T=\begin{pmatrix} 3 & -4 & -1 \\ 0 & 2 & -1 \\ -3 & 3 & 3 \end{pmatrix}.$$

$$A^{-1} = \frac{1}{3}\begin{pmatrix} 3 & -4 & -1 \\ 0 & 2 & -1 \\ -3 & 3 & 3 \end{pmatrix}.$$

Therefore the solution is:

$$x = \frac{1}{3}\begin{pmatrix} 3 & -4 & -1 \\ 0 & 2 & -1 \\ -3 & 3 & 3 \end{pmatrix}\begin{pmatrix} 1570 \\ 740 \\ 880 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 870 \\ 600 \\ 150 \end{pmatrix} = \begin{pmatrix} 290 \\ 200 \\ 50 \end{pmatrix}.$$

Answer:

290 transistors, 200 resistors, and 50 computer chips can be produced.