

Answer on Question #66289 – Math – Calculus

Question

If $f(x, y) = \frac{x^{\frac{1}{4}+y^{\frac{1}{4}}}}{x^{\frac{1}{5}+y^{\frac{1}{5}}}}$, then show that $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = \frac{1}{20} f(x, y)$ stating the results used.

Solution

We need to find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$:

$$\frac{\partial f}{\partial x} = \frac{1}{4(\sqrt[5]{x} + \sqrt[5]{y})x^{\frac{3}{4}}} - \frac{\sqrt[4]{x} + \sqrt[4]{y}}{5(\sqrt[5]{x} + \sqrt[5]{y})^2 x^{\frac{4}{5}}};$$

$$\frac{\partial f}{\partial y} = \frac{1}{4(\sqrt[5]{x} + \sqrt[5]{y})y^{\frac{3}{4}}} - \frac{\sqrt[4]{x} + \sqrt[4]{y}}{5(\sqrt[5]{x} + \sqrt[5]{y})^2 y^{\frac{4}{5}}}.$$

Now we can find $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$:

$$\begin{aligned} x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} &= x \left(\frac{1}{4(\sqrt[5]{x} + \sqrt[5]{y})x^{\frac{3}{4}}} - \frac{\sqrt[4]{x} + \sqrt[4]{y}}{5(\sqrt[5]{x} + \sqrt[5]{y})^2 x^{\frac{4}{5}}} \right) + y \left(\frac{1}{4(\sqrt[5]{x} + \sqrt[5]{y})y^{\frac{3}{4}}} - \frac{\sqrt[4]{x} + \sqrt[4]{y}}{5(\sqrt[5]{x} + \sqrt[5]{y})^2 y^{\frac{4}{5}}} \right) = \\ &= \frac{x^{\frac{1}{4}}}{4(\sqrt[5]{x} + \sqrt[5]{y})} - x^{\frac{1}{5}} \frac{\sqrt[4]{x} + \sqrt[4]{y}}{5(\sqrt[5]{x} + \sqrt[5]{y})^2} + \frac{y^{\frac{1}{4}}}{4(\sqrt[5]{x} + \sqrt[5]{y})} - y^{\frac{1}{5}} \frac{\sqrt[4]{x} + \sqrt[4]{y}}{5(\sqrt[5]{x} + \sqrt[5]{y})^2} = \\ &= \frac{x^{\frac{1}{4}+y^{\frac{1}{4}}}}{4(\sqrt[5]{x} + \sqrt[5]{y})} - \frac{\sqrt[4]{x} + \sqrt[4]{y}}{5(\sqrt[5]{x} + \sqrt[5]{y})^2} \left(x^{\frac{1}{5}} + y^{\frac{1}{5}} \right) = \frac{x^{\frac{1}{4}+y^{\frac{1}{4}}}}{4(x^{\frac{1}{5}+y^{\frac{1}{5}}})} - \frac{x^{\frac{1}{4}+y^{\frac{1}{4}}}}{5(x^{\frac{1}{5}+y^{\frac{1}{5}}})} = \frac{1}{20} \frac{x^{\frac{1}{4}+y^{\frac{1}{4}}}}{(x^{\frac{1}{5}+y^{\frac{1}{5}}})} = \frac{1}{20} f(x, y). \end{aligned}$$