

Answer on Question #65904 – Math – Trigonometry

Question

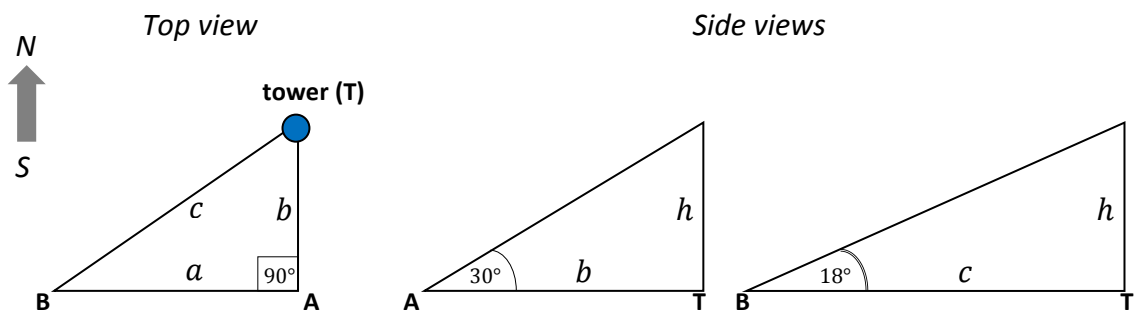
There is a tower the angle of elevation from the point 'A' which is situated South of the tower is 30° . And angle of elevation of the tower from point 'B' which is to the West of point 'A' is 18° .

If $AB = a$

Find the height of tower using 'a'. $(\sin 18^\circ = (\sqrt{5}-1)/4)$.

Solution

Consider these pictures:



Let h be the height of tower, $|AT| = b$, $|BT| = c$.

According to Pythagorean theorem, $a^2 + b^2 = c^2$.

At the same time $\cot 30^\circ = \frac{b}{h}$ and $\cot 18^\circ = \frac{c}{h}$. Therefore $b = h \cot 30^\circ$ and $c = h \cot 18^\circ$.

$$a^2 + (h \cot 30^\circ)^2 = (h \cot 18^\circ)^2 \Rightarrow a^2 = h^2(\cot^2 18^\circ - \cot^2 30^\circ) \Rightarrow$$

$$h = \frac{a}{\sqrt{\cot^2 18^\circ - \cot^2 30^\circ}} = \frac{a}{\sqrt{\cot^2 18^\circ - (\sqrt{3})^2}}$$

$$\cot^2 18^\circ = \frac{1}{(\sin 18^\circ)^2} - 1 = \frac{1}{\left(\frac{\sqrt{5}-1}{4}\right)^2} - 1 = \frac{16}{(\sqrt{5}-1)^2} - 1 = \frac{16 - (\sqrt{5}-1)^2}{(\sqrt{5}-1)^2} = \frac{10 + 2\sqrt{5}}{(\sqrt{5}-1)^2}$$

$$h = \frac{a}{\sqrt{\frac{10 + 2\sqrt{5}}{(\sqrt{5}-1)^2} - 3}} = \frac{a(\sqrt{5}-1)}{\sqrt{10 + 2\sqrt{5} - 3(\sqrt{5}-1)^2}} = \frac{a(\sqrt{5}-1)}{\sqrt{8(\sqrt{5}-1)}} = \frac{\sqrt{\sqrt{5}-1}}{2\sqrt{2}} a$$

Answer: $\frac{\sqrt{\sqrt{5}-1}}{2\sqrt{2}} a$