Answer on Question #65901 – Math – Geometry

Question

For what value of x will the angle between the lines with direction ratios (x, 2, 4) and (1, 0, 1) be 45°?

Solution

Denote $\boldsymbol{a}=(a_1, a_2, a_3) =$(x, 2, 4), $\boldsymbol{b}=(b_1, b_2, b_3) =$(1, 0, 1).

According to the algebraic definition of the dot product (see [1], p.4)

$$\boldsymbol{a}\,\boldsymbol{b} = a_1 b_1 + a_2 b_1 + a_3 b_3.$$

The angle between vectors $\boldsymbol{a}$, $\boldsymbol{b}$ is defined by (see [1], p.6)

$$\cos 45^0 = \frac{\boldsymbol{a}\cdot \boldsymbol{b}}{||\boldsymbol{a}||\cdot||\boldsymbol{b}||}.$$

Using that cos 45°=$\sqrt{2}$/2 and the previous formulas, we get

$a_1 b_1 + a_2 b_1 + a_3 b_3 = ||\boldsymbol{a}|| \cdot ||\boldsymbol{b}|| \cdot \cos 45^0,$

$x \cdot 1 + 2 \cdot 0 + 4 \cdot 1 = \sqrt{x^2 + 4 + 16} \cdot \sqrt{1 + 1} \cdot \frac{\sqrt{2}}{2},$

$x + 4 = \sqrt{x^2 + 20},$

$(x + 4)^2 = \left(\sqrt{x^2 + 20}\right)^2,$

$x^2 + 8x + 16 = x^2 + 20,$

$8x = 20 - 16,$

$8x = 4,$

$x = \frac{4}{8},$

$x = \frac{1}{2}.$

Answer: $x = $ ½.

References:

[1] S. Lipschutz; M. Lipson (2009). Linear Algebra (Schaum's Outlines) (4th ed.). McGraw Hill. ISBN 978-0-07-154352-1.