Problem #6579.The growth of a \$40 000 investment is shown in the table.

Time (years) Value (\$) 0 40 000
1 42 600
2 45 369
3 48 318
4 51 459
5 54 803
Use the exponential regression function to get the function. Round b to the nearest dollar and a to the nearest thousandth.

Solution We are to find such a and b, such that the following equality $V \approx b \cdot a^T$ is the most "precise". This equality is equivalent to $\log V \approx \log b + T \log a$(we can think about this equality as about standard L_2 Gaussian regression $\log V = \log b + T \log a + \varepsilon$, where ε are Gaussian errors). Using standard formulas for the estimation of intercept and slope: $\widehat{\log a} = \dfrac{cov(\log V, T)}{S^2(T)} = 0.03$ and $\widehat{\log b} = \overline{\log V} - \overline{T} \cdot \widehat{\log a} = 4.6$ (here $\widehat{A}$ denotes mean of the sample A, $cov(A, B)$ denotes the covariance of samples A and B). Hence $a = 10^{0.03} \approx 1.0715$ and $b \approx 10^{4.6} \approx 39810.71$.

Answer $V = 39810.71 \cdot 1.0715^T$ or if we round $V = 39811 \cdot 1.072^T$.