Answer on Question #65691 – Math – Linear Algebra

Question

Let V be the vector space of 2×2 matrices over R. Check whether the subsets

$W1 = \{ (a, 1), (0, -a) \mid a \in R\}$ and $W2 = \{ (a, -a), (0, b) \mid a, b \in R\}$ are subspaces over R. For those sets which are subspaces, find their dimension and a basis over R.

Solution

We use the criterion subspace linear space.

Let $A = \begin{pmatrix} a & 1 \\ 0 & -a \end{pmatrix} \in W_1$ and $B = \begin{pmatrix} b & 1 \\ 0 & -b \end{pmatrix} \in W_1$.

Because $A + B = \begin{pmatrix} a+b & 2 \\ 0 & -a-b \end{pmatrix} \notin W_1$, then W_1 is not a subspace.

Let $A = \begin{pmatrix} a & -a \\ 0 & b \end{pmatrix} \in W_2$, $B = \begin{pmatrix} c & -c \\ 0 & d \end{pmatrix} \in W_2$ and $\alpha, \beta \epsilon \mathbb{R}$.

Because $\alpha A + \beta B = \begin{pmatrix} \alpha a + \beta c & -(\alpha a + \beta c) \\ 0 & \alpha b + \beta d \end{pmatrix} \in W_2$, then W_2 is a subspace.

Using the definitions of basis and dimension one gets that the set of vectors $\left\{ \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ is the basis of W_2 and $dim_{\mathbb{R}} W_2 = 2$.

Answer: W_1 is not a subspace; W_2 is a subspace; $\left\{ \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ is the basis of W_2; $dim_{\mathbb{R}} W_2 = 2$.