Answer on Question#65569 – Math – Statistics and Probability

Question. True or false? Give reason.

If correlation coefficient between x and y is 0.62, then correlation coefficient between u and v will be 0.62, where $u = 5 + 6x$ and $v = 7 - 3y$.

Solution. Let us start from definition of correlation coefficient

(see https://en.wikipedia.org/wiki/Pearson_correlation_coefficient):

$\rho_{x,y} = \frac{cov(x,y)}{\sigma_x \sigma_y}$. Then $\rho_{u,v} = \frac{cov(u,v)}{\sigma_u \sigma_v}$. Since $\begin{cases} u = 5 + 6x \\ v = 7 - 3y \end{cases}$, using the properties of variance

(see https://en.wikipedia.org/wiki/Variance#Properties) we get:

$Var(u) = Var(5 + 6x) = 36Var(x);\ Var(v) = Var(7 - 3y) = 9Var(y)$. Then

$\sigma_u = \sqrt{Var(u)} = \sqrt{36Var(x)} = 6\sqrt{Var(x)} = 6\sigma_x$. Similarly $\sigma_v = 3\sigma_y$.

$cov(x, y) = E\big[\big(x - E(x)\big)\big(y - E(y)\big)\big]$

(see https://en.wikipedia.org/wiki/Pearson_correlation_coefficient). Then

$cov(u, v) = E\big[\big(u - E(u)\big)\big(v - E(v)\big)\big]$. Using the properties of mathematical expectation

(see https://en.wikipedia.org/wiki/Expected_value#Properties) we get:

$u - E(u) = 5 + 6x - E(5 + 6x) = 5 + 6x - \big(5 + 6E(x)\big) = 6\big(x - E(x)\big)$. Similarly

$v - E(v) = 7 - 3y - E(7 - 3y) = 7 - 3y - \big(7 - 3E(y)\big) = -3\big(y - E(y)\big)$. Then we have:

$\rho_{u,v} = \frac{E\big[6\big(x - E(x)\big) \cdot (-3)\big(y - E(y)\big)\big]}{6\sigma_x \cdot 3\sigma_y} = \frac{-18E\big[\big(x - E(x)\big)\big(y - E(y)\big)\big]}{18\sigma_x \sigma_y} = -\rho_{x,y} = -0.62 \neq 0.62.$

The statement is false.

Answer. False.