Answer on Question #65544, Math / Statistics and probability
Find all the basic solutions of the following system of equations:
$$\begin{cases} x_1 + 2x_2 + x_3 = 14 \\ 3x_1 + x_2 + x_3 = 12 \end{cases}$$
Which of these solutions are basic feasible solutions?
Solution
Definition Basic Solution: A solution obtained by setting exactly n−m variables to zero provided the determinant formed by the columns associated to the remaining m variables is non-zero is called Basic Solution.
$$\begin{cases} x_1 + 2x_2 + x_3 = 14 \\ 3x_1 + x_2 + x_3 = 12 \end{cases}$$

$$[A, b] = \begin{bmatrix} 1 & 2 & 1 & 14 \\ 3 & 1 & 1 & 12 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - (3)R_1$$

$$\begin{bmatrix} 1 & 2 & 1 & 14 \\ 0 & -5 & -2 & -30 \end{bmatrix}$$

$$R_2 \rightarrow R_2/(-5)$$

$$\begin{bmatrix} 1 & 2 & 1 & 14 \\ 0 & 1 & 2/5 & 6 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - (2)R_2$$

$$\begin{bmatrix} 1 & 0 & 1/5 & 2 \\ 0 & 1 & 2/5 & 6 \end{bmatrix}$$

Basic solutions
$x_1 = 2, \qquad x_2 = 6, \qquad x_3 = 0$
$x_1 = -1, \qquad x_2 = 0, \qquad x_3 = 15$
$x_1 = 0, \qquad x_2 = 2, \qquad x_3 = 10$
x is a feasible basic solution if x is basic and $x \geq 0$.
Therefore
$x = (2, 6, 0)^T$ is a basic feasible solution
$x = (-1, 0, 15)^T$ is a basic feasible solution but not feasible
$x = (0, 2, 10)^T$ is a basic feasible solution