Answer on Question #65433 – Math | Geometry

1. show that points A(2,-3), B(5,0), C(2,3), and D(-1,0), are the vertices of a square.

Lets calculate length of diagonals BD and AC;$AC = \sqrt{(2-2)^2 + (3+3)^2} = 6;$

$BD = \sqrt{(5+1)^2 + 0} = 6$ As, two diagonals are equal: quadrangle **is square**;

2. the distance (x,1) is 2 square root of 5 units from (2, 3), find x

$$\sqrt{(x-2)^2 + (1-3)^2} = 2\sqrt{5};$$

$$(x-2)^2 = 16;$$

$$x - 2 = \pm 4;$$

$$\boldsymbol{x = -2, x = 6}$$

3. what are the coordinates of the point 3 units from the Y-axis and at distance square root 5 from (5,3)?

As the point is 3 units from Y-axis: its x component equals 3 or -3;

$$\sqrt{(\pm 3 - 5)^2 + (y-3)^2} = \sqrt{5}$$

$8^2 + (y-3)^2 = 5$ $\quad$ $2^2 + (y-3)^2 = 5$

$y \in \emptyset$ $\quad$ $(y-3)^2 = 1$

$$y - 3 = \pm 1$$

$$y = 4, y = 2$$

Answer: **(3,4) or (3,2)**

4. what is the equation of a line through A(7,-3) and perpendicular to the line whose inclination is Arctan 2/3.

Let $y = k_1 x + b_1 - equation\ of\ first\ line$

inclination: arctan2/3 => $k_1 = \frac{2}{3};$

Let $y = k_2 x + b_2 - equation\ of\ second\ \ line,\quad if\ line1\ is\ perpendicular\ to\ line2;\quad k_2 = -\frac{1}{k_1} = -\frac{3}{2};$

$$y = -\frac{3}{2}x + b_2;\ \mathrm{A} \in \mathrm{line2};\quad \mathrm{y}_A = -\frac{3}{2}x_A + b_2;$$

$$-3 = -\frac{3}{2} * 7 + b_2;$$

$$b_2 = \frac{15}{2};$$

So, equation of line2: $\mathbf{y} = -\frac{\mathbf{3}}{\mathbf{2}}\mathbf{x} + \frac{\mathbf{15}}{\mathbf{2}}$

5. show that lines 2x + 3Y - 2 =0, 3X - 2Y + 23 =0 and X - 5Y = 12 = 0 are the sides of an isosceles triangle.

$$Let\ vector\ \vec{a},\vec{b},\vec{c}, parallel\ to\ line\ 1{,}2{,}3;\ \ \vec{a} = (2,3);\ \vec{b} = (3,-2); \vec{c} = (1,-5)$$

$$Lets\ find\ angles\ between\ vector\ \vec{a},\vec{c}\ \ and\ \vec{b},\vec{c};$$

$$\varphi(\vec{a},\vec{c}) = \frac{\vec{a}\vec{c}}{|\vec{a}||\vec{c}|} = \frac{2*1-5*3}{13\sqrt{2}} = -\frac{1}{\sqrt{2}};\ \varphi = 135°;$$

$$\varphi(\vec{b},\vec{c}) = \frac{\vec{b}\vec{c}}{|\vec{b}||\vec{c}|} = \frac{3*1+2*5}{13\sqrt{2}} = \frac{1}{\sqrt{2}};\ \varphi = 45°;$$

Angle between vectors **a** and **c** = 135^o (45 ^{o})
Angle between vectors **b** and **c** = 45 ^{o}
As angles between vectors are same, angles between lines are same, so the triangle is **isosceles**;