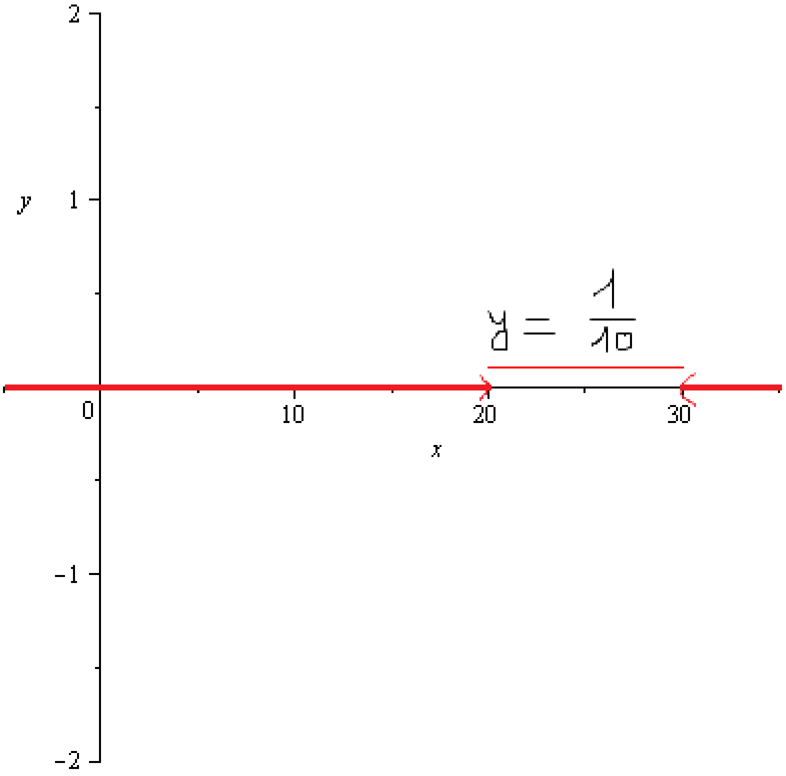
Answer on Question#65392 – Math – Statistics and Probability

Question. Consider a random variable X having the uniform density function $f(x)$ with $a = 20$ and $b = 30$.

i) Define and graph the density function $f(x)$.

Solution. Using the definition of uniform continuous density function (see https://en.wikipedia.org/wiki/Uniform_distribution_(continuous)) we obtain:

$$f(x) = \begin{cases} \frac{1}{30-20}, 20 \le x \le 30 \\ 0, x < 20 \; or \; x > 30 \end{cases} = \begin{cases} \frac{1}{10}, 20 \le x \le 30 \\ 0, x < 20 \; or \; x > 30 \end{cases}.$$



Answer. $f(x) = \begin{cases} \frac{1}{10}, 20 \le x \le 30 \\ 0, x < 20 \; or \; x > 30 \end{cases}.$

ii) Verify that $f(x)$ is a probability density function.

Proof. Obviously $f(x) \ge 0$ for all $x \in \mathbb{R}$. Now we must check that $\int_{-\infty}^{\infty} f(x)dx = 1$ (see https://onlinecourses.science.psu.edu/stat414/node/97, Definition).

Obviously $\int_{-\infty}^{20} 0dx = \int_{30}^{\infty} 0dx = 0$. Then

$\int_{-\infty}^{\infty} f(x)dx = \int_{20}^{30} \frac{1}{10} dx$. Using the linearity and Second fundamental theorem of calculus

(see https://en.wikipedia.org/wiki/Integral) we get:

$\int_{20}^{30} \frac{1}{10} dx = \frac{1}{10} \int_{20}^{30} dx = \frac{1}{10} x|_{x=20}^{30} = \frac{1}{10} \cdot (30 - 20) = \frac{1}{10} \cdot 10 = 1.$

Indeed $f(x)$ is a probability density function.

iii) Find $P(22 \leq X \leq 30)$.

Solution. Using https://onlinecourses.science.psu.edu/stat414/node/97, Definition, (3) we get:

$P(22 \leq X \leq 30) = \int_{22}^{30} f(x)dx = \int_{22}^{30} \frac{1}{10} dx = \frac{1}{10} \cdot (30 - 22) = \frac{8}{10} = \frac{4}{5} = 0.8.$

Answer. 0.8.

iv) Find $P(X = 25)$.

Solution. Similarly to https://onlinecourses.science.psu.edu/stat414/node/97 (see example with $P\left(X = {}^{1}/_{2}\right)$) we have:

$P(X = 25) = \int_{25}^{25} \frac{1}{10} dx = \frac{1}{10} \cdot (25 - 25) = \frac{1}{10} \cdot 0 = 0.$

Answer. 0.