Answer on Question #65385 - Math - Statistics and Probability

Question: Let $x_1, x_2, \dots, x_n$ be a random sample from a distribution with density function

$$f(x|\alpha) = \begin{cases} \frac{1}{\alpha}, & 0 \le x \le \alpha; \\ 0, & otherwise \end{cases}$$

Obtain the maximum likelihood estimator of α.

Solution: Let $x_{(1)} \le x_{(2)} \le \dots \le x_{(n)}$ be the order statistics. The likelihood function is given by

$$L(\alpha; x_1, x_2, \dots, x_n) = f(x_1, x_2, \dots, x_n|\alpha) = \prod_{k=1}^{n} f(x_k|\alpha) = \prod_{k=1}^{n} \frac{1}{\alpha} = \alpha^{-n}$$

for $0 \le x_{(1)}$ and $\alpha \ge x_{(n)}$, and 0 otherwise.

Then, the log-likelihood function is equal to

$\ln L(\alpha; x_1, x_2, \dots, x_n) = -n \ln \alpha$ for $0 \le x_{(1)}$ and $\alpha \ge x_{(n)}$, and 0 otherwise.

Now taking the derivative of the log-likelihood wrt α gives:

$$\frac{\partial \ln L(\alpha; x_1, x_2, \dots, x_n)}{\partial \alpha} = -\frac{n}{\alpha} < 0.$$

So, we can say that $L(\alpha; x_1, x_2, \dots, x_n)$ is a decreasing function for $\alpha \ge x_{(n)}$. Therefore, $L(\alpha; x_1, x_2, \dots, x_n)$ is maximized at $\alpha = x_{(n)}$. Hence, the maximum likelihood estimator for α is given by

$\hat{\alpha} = x_{(n)} = \max\{x_1, x_2, \dots, x_n\}$.

Answer: $\hat{\alpha} = x_{(n)} = \max\{x_1, x_2, \dots, x_n\}$.

For maximum likelihood estimation (MLE) see
https://en.wikipedia.org/wiki/Maximum_likelihood_estimation or
https://ocw.mit.edu/courses/mathematics/18-443-statistics-for-applications-fall-2006/lecture-notes/lecture2.pdf or
https://www.projectrhea.org/rhea/index.php/Maximum_Likelihood_Estimation_Analysis_for_various_Probability_Distributions