Answer on Question#65286, Math / Differential Equations

6

Question

Calculate the surface integral of $\oiint_S \vec{F} d\vec{s}$, where $\vec{F} = 2(x, y, z)$, and where S is the surface of the cube define by the relations $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$.

Solution

According to divergence theorem:

$$\oiint_S \vec{F} d\vec{s} = \iiint_V (\vec{\nabla} \cdot \vec{F}) dV$$

$$\vec{\nabla} \cdot \vec{F} = 2\left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}\right) = 2(1 + 1 + 1) = 6$$

$$\iiint_V (\vec{\nabla} \cdot \vec{F}) dV = \iiint_V 6 dV = 6 \iiint_V dV = 6V$$

Volume of the unit cube is 1, hence

$$\oiint_S \vec{F} d\vec{s} = 6$$