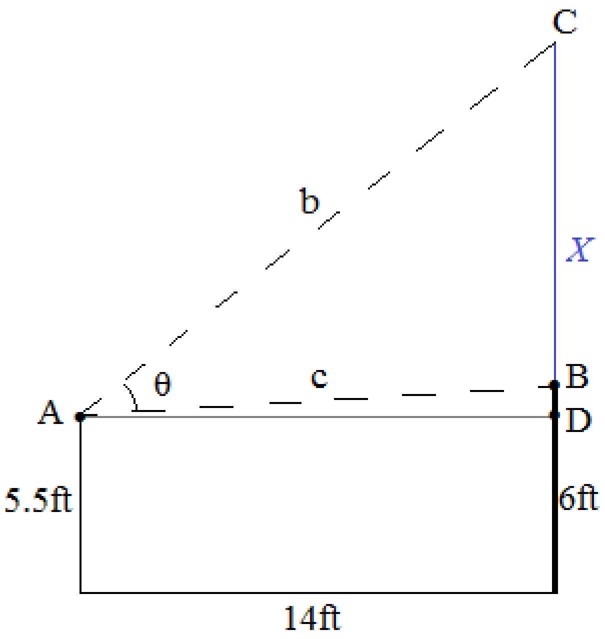
Answer on Question #65271, Math / Geometry

You want to determine the height of the screen at a drive-in movie theater. You use a cardboard square to line up the top and bottom of the screen structure. The vertical distance from the ground to your eye is 5.5 ft and the horizontal distance from you to the screen is 14 ft. The bottom of the screen is 6 feet from the ground. Approximate the height of the screen to the nearest tenth.



Let X = the height of the screen.
ΔABC Sine rule

$$\frac{X}{\sin\theta} = \frac{c}{\sin C}$$

Right triangle ΔABD Pythagorean Theorem

$$c^2 = 14^2 + (6 - 5.5)^2 = 196.25$$

Right triangle ΔACD Pythagorean Theorem

$$b^2 = 14^2 + (X + 6 - 5.5)^2 = X^2 + X + 196.25$$

$$\sin C = \frac{14}{b}$$

Then

$$X = c\frac{\sin\theta}{\sin C} = bc\frac{\sin\theta}{14}$$

$$14X = \sqrt{X^2 + X + 196.25}\sqrt{196.25}\sin\theta$$

$$196X^2 = 196.25(X^2 + X + 196.25)\sin^2\theta$$

$$\frac{196}{196.25\sin^2\theta}X^2 = X^2 + X + 196.25$$

$$0 < \theta < 90, \frac{196}{196.25\sin^2\theta} > 1 \Rightarrow 196 > 196.25\sin^2\theta \Rightarrow 0 < \sin\theta < \frac{14}{\sqrt{196.25}}$$

$$1 + \cot^2\theta = \frac{1}{\sin^2\theta} > \frac{196.25}{196} \Rightarrow \cot^2\theta > \frac{196.25}{196} - 1 \Rightarrow \cot^2\theta > \frac{0.25}{196} \Rightarrow$$

$=> \cot\theta > \frac{1}{28} => 28\cot\theta > 1$

$\left(\frac{196}{196.25\sin^2\theta} - 1\right)X^2 - X - 196.25 = 0$

$D = 1 - 4\left(\frac{196}{196.25\sin^2\theta} - 1\right)(-196.25) = 1 - 785 + \frac{784}{\sin^2\theta} = 784\frac{1-\sin^2\theta}{\sin^2\theta} =$
$= 784\cot^2\theta$

$X = \frac{1 + 28\cot\theta}{2\left(\frac{196}{196.25\sin^2\theta} - 1\right)} = \frac{\sin^2\theta}{2}\cdot\frac{1 + 28\cot\theta}{\frac{196}{196.25} - \sin^2\theta} \approx \frac{\sin^2\theta}{2}\cdot\frac{1 + 28\cot\theta}{1 - \sin^2\theta} \approx$
$\approx \frac{1}{2}\tan^2\theta\,(1 + 28\cot\theta)\,ft, \cot\theta > \frac{1}{28}.$

Answer:

the height of the screen $\approx \frac{1}{2}\tan^2\theta\,(1 + 28\cot\theta)\,ft, \cot\theta > \frac{1}{28}.$